TEST PREP

44. D: $\frac{10}{5} = 2$; $\frac{20}{10} = 2$; $\frac{40}{20} =2$; there is a common ratio.

45. J; since $r = -4$ and $a_1 = 2$,
$\left(\frac{-8}{2} = -4; \frac{32}{-8} = -4; \frac{-128}{32} = -4\right)$
$a_n = 2\,(-4)^{n-1}$

46. C; $r = 2$ and $A_1 = 55$
$A_n = A_a r^{n-1}$
$A_7 = A_1\, r^6$
$A_7 = 3520$ Hz

CHALLENGE AND EXTEND

47. $\frac{x^2}{x} = x$; $\frac{x^3}{x^2} = x$
$r = x$ and $a_1 = x$;
$a_4 = x\,(x)^3 = x^4$
$a_5 = x\,(x)^4 = x^5$
$a_6 = x\,(x)^5 = x^6$

48. $\frac{6x^3}{2x^2} = 3x$; $\frac{18x^4}{6x^3} = 3x$
$r = 3x$ and $a_1 = 2x^2$;
$a_4 = 2x^2\,(3x)^3 = 54x^5$
$a_5 = 2x^2\,(3x)^4 = 162x^6$
$a_6 = 2x^2\,(3x)^5 = 486x^7$

49. $\frac{1}{y^2} \div \frac{1}{y^3} = y$; $\frac{1}{y} \div \frac{1}{y^2} = y$
$r = y$ and $a_1 = \frac{1}{y^3}$
$a_4 = \frac{1}{y^3}\,(y)^3 = 1$
$a_5 = \frac{1}{y^3}\,(y)^4 = y$
$a_6 = \frac{1}{y^3}\,(y)^5 = y^2$

50. $\frac{1}{x+1} \div \frac{1}{(x+1)^2} = x + 1$; $1 \div \frac{1}{x+1} = x + 1$
$r = x + 1$ and $a_1 = \frac{1}{(x+1)^2}$
$a_4 = \frac{1}{(x+1)^2}\,(x + 1)^3 = x + 1$
$a_5 = \frac{1}{(x+1)^2}\,(x + 1)^4 = (x + 1)^2$
$a_6 = \frac{1}{(x+1)^2}\,(x + 1)^5 = (x + 1)^3$

51. $a_{10} = a_1 r^9$
$a_1 = \frac{a_{10}}{r^9}$
$a_1 = \frac{0.78125}{(-0.5)^9}$
$a_1 = -400$

52. No; each term of the sequence is found by multiplying the previous term by the common ratio $\frac{1}{2}$. $\frac{1}{2}$ of any positive number is always another positive (nonzero) number.

53. $a_n = a_1 r^{n-1}$
$r^{n-1} = \frac{a_n}{a_1}$
$(0.4)^{n-1} = \frac{0.057344}{14}$
$(0.4)^{n-1} = (0.4)^6$
Then, $n - 1 = 6$
$n = 7$

54. Susanna assumed the sequence was geometric with $r = 2$. She used the formula to find $a_8 = 128$. Paul did not assume the sequence was geometric. Instead, he noticed a pattern of “add 1, add 2, and so on.” He continued this pattern by adding 3, adding 4, etc., until he got the 8th term of 29. Both could be considered correct because it was not specified what type of sequence was given.

9-2 EXPONENTIAL FUNCTIONS

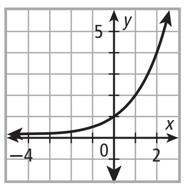
CHECK IT OUT!

1. $f(x) = 8(0.75)^x$
$f(3) = 8(0.75)^3$
$f(3) = 8(0.421875)$
$f(3) = 3.375$ in.

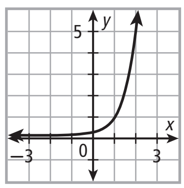
2a. No; as the x-values change by a constant amount, the y-values are not multiplied by a constant amount.

b. Yes; as the x-values change by a constant amount, the y-values are multiplied by a constant amount.

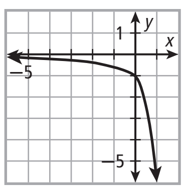
3a. $y = 2^x$



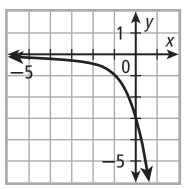
b. $y = 0.2(5)^x$



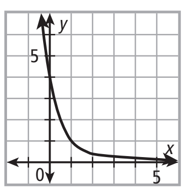
4a. $y = -6^x$



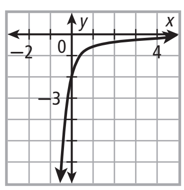
b. $y = -3(3)^x$



5a. $y = 4\left(\frac{1}{4}\right)^x$



b. $y = -2(0.1)^x$

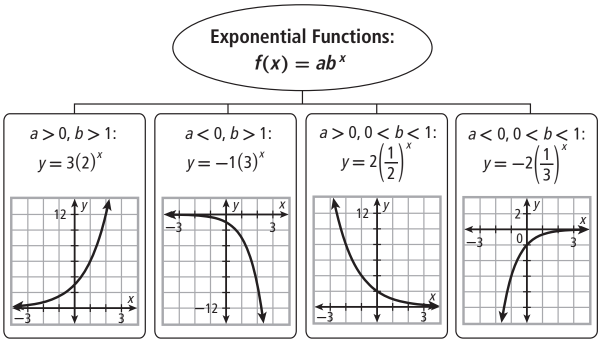


6. $f(x) = 12{,}330(0.869)^x$
$2000 = 12{,}330(0.869)^x$
$x = log_{0.869}\left(\dfrac{2\,000}{12{,}330}\right)$
$x \approx 13$; after about 13 yrs

THINK AND DISCUSS

1. Possible answer: Make a table of values. Use x-values that change by the same amount each time as you move down the column. Then divide each y-value, starting with the second row, by the y-value before it. The quotient is the common ratio.

2.



EXERCISES

GUIDED PRACTICE

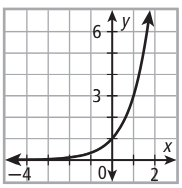
1. No; there is no variable in the exponent.

2. $f(x) = 50{,}000(0.975)^x$
$f(200) = 50{,}000(0.975)^{200}$
$f(200) = 316$; 316 lumens/m^2

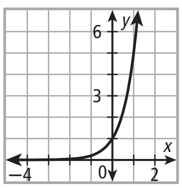
3. No; as the x-values increase by a constant value, the y-values are not multiplied by a constant value.

4. Yes; as the x-values increase by a constant value, the y-values are multiplied by a constant value.

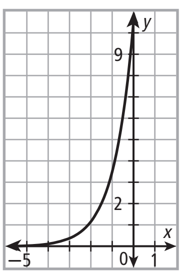
5. $y = 3^x$



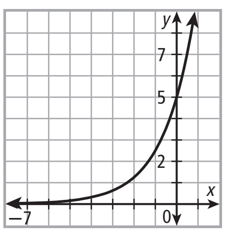
6. $y = 5^x$



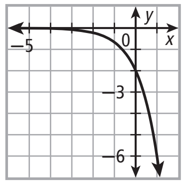
7. $y = 10(3)^x$



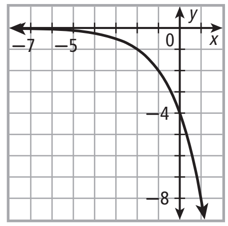
8. $y = 5(2)^x$



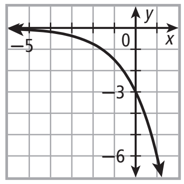
9. $y = -2(3)^x$



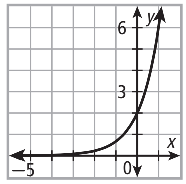
10. $y = -4(2)^x$



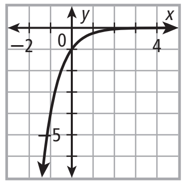
11. $y = -3(2)^x$



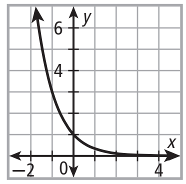
12. $y = 2(3)^x$



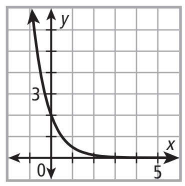
13. $y = -\left(\dfrac{1}{4}\right)^x$



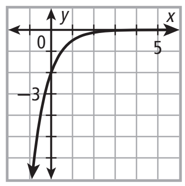
14. $y = \left(\dfrac{1}{3}\right)^x$



15. $y = 2\left(\dfrac{1}{4}\right)^x$



16. $y = -2(0.25)^x$



17. $f(x) = 57.8(1.02)^x$
$200{,}000{,}000 = 57.8(1.02)^x$
$x \approx 63$;
about 2023 (63 years after 1960)

PRACTICE AND PROBLEM SOLVING

18. $f(x) = 27\left(\dfrac{2}{3}\right)^x$
$f(4) = 27\left(\dfrac{2}{3}\right)^4$
$f(4) = 27\left(\dfrac{16}{81}\right)$
$f(4) = 5\dfrac{1}{3}$; $5\dfrac{1}{3}$ ft

19. $y = 334(0.976)^x$
for $x = 6$,
$y = 334(0.976)^6$
$y \approx 289$; 289 ft

20. $y = 1.3(1.41)^x$
for $x = 15$,
$y = 1.3(1.41)^{15}$
$y \approx 225.02$; 225.02 in./min

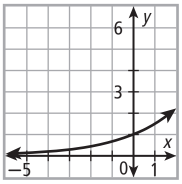
21. Yes; as the x-values change by a constant amount, the y-values are multiplied by a constant amount.

22. No; as the x-values change by a constant amount, the y-values are not multiplied by a constant amount.

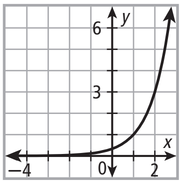
23. No; as the x-values change by a constant amount, the y-values are not multiplied by a constant amount.

24. Yes; as the x-values change by a constant amount, the y-values are multiplied by a constant amount.

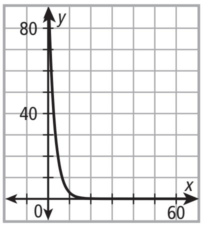
25. $y = 1.5^x$



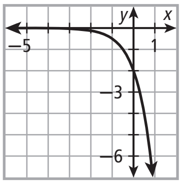
26. $y = \frac{1}{3}(3)^x$



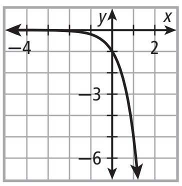
27. $y = 100(0.7)^x$



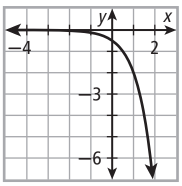
28. $y = -2(4)^x$



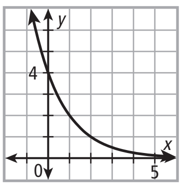
29. $y = -1(5)^x$



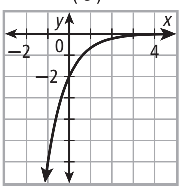
30. $y = -\frac{1}{2}(4)^x$



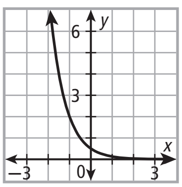
31. $y = 4\left(\frac{1}{2}\right)^x$



32. $y = -2\left(\frac{1}{3}\right)^x$



33. $y = 0.5(0.25)^x$



34. $f(x) = 42(1.41)^x$
$1{,}000 = 42(1.41)^x$
$x \approx 9$; about 2009

35. $y = (3.1x + 7)^2$ is not exponential since there is no variable in the exponent.

For $y = \left(\frac{1}{5}\right)(6)^x$, $y = 7.2$ for $x = 2$ and $y = 43.2$ for $x = 3$, hence $y = \left(\frac{1}{5}\right)(6)^x$ does not generate 38.4.

For $y = 4.8(2)^x$, $y = 38.4$ for $x = 3$; ans. $y = 4.8(2)^x$

36a. $f(x) = 20(1.2)^x$
$f(2) = 20(1.2)^2$
$f(2) = 28.8$; \$28.80

b. $f(x) = 20(1.2)^x$
$100 \geq 20(1.2)^x$
$x \geq 9$; after 9 weeks

c. $f(x) = 20(1.2)^x$
$f(0) = 20(1.2)^0$
$f(0) = 20$; \$20

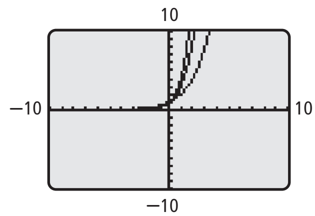
d. $\text{increase} = \dfrac{f(n+1)}{f(n)} - 1$

$\text{increase} = \dfrac{20(1.2)^{n+1}}{20(1.2)^n} - 1$

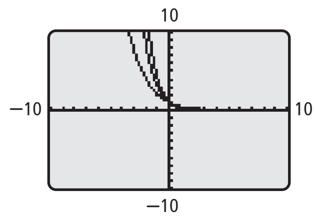
increase = .2; .2 or 20%

37. If the value of b were 1, the function would be constant. If the value of a were 0, the function would be the constant function $y = 0$.

38. Possible answer: The graphs have the same basic shape and the same y-intercept; each graph is steeper than the one before it.



39. Possible answer: The graphs have the same basic shape and the same y-intercept; each graph is steeper than the one before it.



40. $f(x) = 4^x$
$f(3) = 4^3$
$f(3) = 64$

41. $f(x) = -(0.25)^x$
$f(1.5) = -(0.25)^{1.5}$
$f(1.5) = -0.125$

42. $f(x) = 0.4(10)^x$
$f(-3) = 0.4(10)^{-3}$
$f(-3) = 0.0004$ or 4×10^{-4}

43a. In 2001, $n = 0$
$C = 2000(1.08)^0$
$C = 2000$; \$2000

b. $\text{increase} = \dfrac{2000(1.08)^{n+1}}{2000(1.08)^n} - 1$

increase = 0.08; 8%

c. For 2006, $n = 5$
$C = 2000(1.08)^5$
$C = 2938.66$; \$2938.66

44. Possible answer: The following table shows how much money you could earn with each plan.

Year	Salary Plan A	Salary Plan B
0	\$0	\$10,000
1	\$20,000	\$20,000
2	\$40,000	\$40,000
3	\$60,000	\$80,000

Choose plan B because plan A doesn't pay anything for the first year and because after 3 years, plan B pays more money.

45. C; the other graphs do not increase exponentially.

46. G; $f(4) = 15(1.4)^2 = 29.4$

47. D; $a_1 = 5$, $r = 5$, hence $a_n = 5(5)^{n-1} = 5^n$

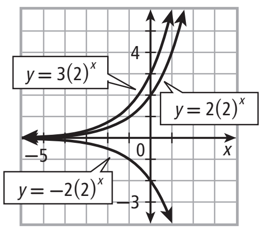
CHALLENGE AND EXTEND

48. $4^x = 64$
$4^x = 4^3$
$x = 3$

49. $\left(\frac{1}{3}\right)^x = \frac{1}{27}$
$3^{-x} = 3^{-3}$
$-x = -3$
$x = 3$

50. $2^x = \frac{1}{16}$
$2^x = \frac{1}{24}$
$2^x = 2^{-4}$
$x = -4$

51. The value of a is the y-intercept.



READY TO GO ON? Section A Quiz

1. $\frac{6}{3} = 2$; $\frac{12}{6} = 2$; $\frac{24}{12} = 2$; the common ratio is 2
next 3 terms: $24(2) = 48$, $48(2) = 96$, and $96(2) = 192$

2. $\frac{2}{-1} = -2$; $\frac{-4}{2} = -2$; $\frac{8}{-4} = -2$;
the common ratio is -2
next 3 terms: $8(-2) = -16$, $(-16)(-2) = 32$, and $32(-2) = -64$

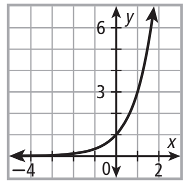
3. $\frac{-1200}{-2400} = \frac{1}{2}$; $\frac{-600}{-1200} = \frac{1}{2}$; $\frac{-300}{-600} = \frac{1}{2}$
next 3 terms: $-300\left(\frac{1}{2}\right) = -150$, $-150\left(\frac{1}{2}\right) = -75$,
and $-75\left(\frac{1}{2}\right) = -37.5$

4. $a_n = a_1r^{n-1}$
$a_8 = (2)(3)^{8-1}$
$a_8 = 4374$

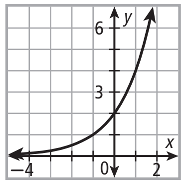
5. $a_1 = 1000$, $r = \frac{4}{5}$
$a_n = a_1r^{n-1}$
$a_7 = 1000\left(\frac{4}{5}\right)^{7-1}$
$a_7 = 262.144$ cm

6. $f(x) = 3(1.1)^x$
$f(4) = 3(1.1)^4$
$f(4) = 4.39$ in

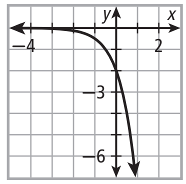
7. $y = 3^x$



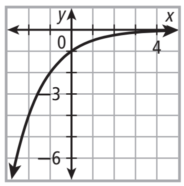
8. $y = 2(2)^x$



9. $y = -2(4)^x$



10. $y = -(0.5)^x$



11. $f(x) = 40(0.8)^x$
$2 = 40(0.8)^x$
$x \approx 14$;
after about 14 h

9-3 EXPONENTIAL GROWTH AND DECAY

CHECK IT OUT!

1. $y = a(1 + r)^t$
$= 1200(1.08)^t$;
In 2006, $y = 1200(1.08)^6 = \$1904.25$

2a. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
$= 1200\left(1 + \frac{0.035}{4}\right)^{4t}$
$= 1200(1.00875)^{4t}$;
After 4 years, $A = 1200(1.00875)^{16} = \$1379.49$

b. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
$= 4000\left(1 + \frac{0.03}{12}\right)^{12t}$
$= 4000(1.0025)^{12t}$
After 8 years, $A = 4000(1.0025)^{96} = \$5083.47$

3. $y = a(1 - r)^t$
$= 48{,}000(1 - 0.03)^t$
$= 48{,}000(0.97)^t$
After 7 years, $y = 48{,}000(0.97)^7 = 38{,}783$

4a. $t = \frac{180 \text{ years}}{30 \text{ years}}$
$= 6$
$A = P(0.5)^t$
$= 100(0.5)^6$
$= 1.5625$ mg