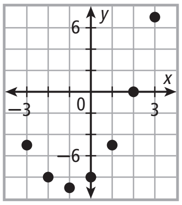
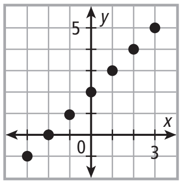
PRACTICE AND PROBLEM SOLVING

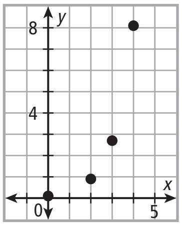
8. quadratic



9. linear



10. exponential



11. Linear, for every constant change of +1 in the x-values, there is a constant change of −1 in the y-values.

12. Quadratic, for every constant change of +1 in the x-values, there is a constant second difference of −2 in the y-values.

13. Exponential, for every constant change of +1 in the x-values, there is a constant ratio of 0.5 in the y-values.

14. The company's sales are increasing by 20% each year; $y = 25{,}000(1.2)^x$; \$154,793.41

15. $l = 6k$; linear with $m = 6$ and $b = 0$

16. Linear; for every weekly interval, the height of the plant has a constant increase of 0.5 inches.

17. Linear; for each successive year, the number of games has a constant change of 0.

18. Quadratic; for each successive time interval, the height of a ball has a constant second difference of −0.28.

19. $y = 0.2(4)^x$

20. $y = -\frac{1}{2}x + 4$

21. linear

22. quadratic

23. Possible answer: (0,3), (1,6), (2,12), (3,24); for a constant change in x of +1, there is a common ratio of 2.

24. Possible answer: the first differences are constant, so there is no need to check the second differences. A linear function would best model the data.

25. Possible answer: make a table of ordered pairs and see whether the y-values show a pattern of constant second differences or constant ratios.

26a. college 1: linear because it has constant changes of \$200 each year; college 2: exponential because it has a constant yearly ratio of 1:1.1.

b. college 1: $y = 200x + 2000$;
college 2: $y = 2000(1.1)^x$

c. Both have the same tuition (\$2000) in 2004.

d. For college 1, \$200 is added each year, so 2000 + 200 = 2200. For college 2, 10% is added each year, so 2000 + (0.1)(2000) = 2200.

27. C; the data is linear since it has a constant change in the y-values for each constant change in the x-values.

28. F; 2% is a common ratio.

29. C; For every constant change of +1 in the x-values, there is a constant change of +2 in the y-values.

CHALLENGE AND EXTEND

30a.

Year	Value (\$)
0	18,000
1	15,120
2	12,700.80
3	10,668.67
4	8961.68

Year 0 is the year when the car is purchased.

b. exponential, for each successive year, the value decreases by 16%, the common ratio.

c. $y = 18{,}000(0.84)^x$

d. $y = 18{,}000(0.84)^{5\frac{1}{2}} = \$6899.36$

e. $y = 18{,}000(0.84)^8 = \$4461.77$

31a. Possible answer: quadratic; the second differences are approximately constant at −2.

b. about 48 kg

c. No; this quadratic model will begin to decrease although the dog's weight will either continue to grow or eventually remain constant.

9-5 COMPARING FUNCTIONS

CHECK IT OUT!

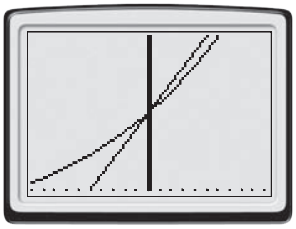
1. Slope: For Dave, use (0, 30) and (1, 42). $\frac{42-30}{1-0} = 12$. For Arturo, use (0, 24) and (1, 32). $\frac{32-24}{1-0} = 8$. Dave is saving at a higher rate (\$12/wk) than Arturo (\$8/wk); y-int.: The y-intercept for Dave is (0, 30) and for Arturo is (0, 24). Dave started with more money (\$30) than Arturo (\$24).

2. Average rate of change of Investment A: use (10, 17.91) and (25, 42.92).
$\frac{42.92-17.91}{25-10} \approx 1.67$; A increased about \$1.67/yr; Average rate of change of Investment B: use (10, 17) and (25, 34). $\frac{34-17}{25-10} \approx 1.13$; B increased about \$1.13/yr.

3. Rosetta's new model:
max ht.: $x = -\frac{b}{2a} = -\frac{1.1}{2(-0.01)} = \frac{1.1}{0.02} = 55$;
$y = -0.01(55)^2 + 1.1(55) = 30.25$ ft;
length: (x–intercepts)
$-0.01x^2 + 1.1x = 0$
$-x(0.01x - 1.1) = 0$

$x = 0$ or $x = 110$
length = 110 ft;
avg. steepness over [0, 20]:
At $x = 0$, $y = -0.01(0)^2 + 1.1(0) = 0$
At $x = 20$, $y = -0.01(20)^2 + 1.1(20) = 18$
Use (0, 0) and (20, 18) = 0.9;
The new model is taller, longer, and steeper over [0, 20] than Marco's.

4. A: $y = 100x + 850$; B: $y = 850(1.08)^x$;



school A's enrollment will exceed school B's at first, but school B will have more students by the end of the 11th yr. After that, school B's enrollment exceeds school A's by ever-increasing amts each yr.

THINK AND DISCUSS

1. Rates of change for quadratic and exponential functions are variable, but rates of change for linear functions are constant.

2. Using an equation, you can find exact values. Using a graph, you can quickly see how variables are related.

3. Possible answers:

Comparing Functions			
Linear to Linear	**Exponential to Exponential**	**Quadratic to Quadratic**	**Linear to Quadratic**
Compare… slopes and y-intercepts	Compare… rates of change over different intervals	Compare… maximums or minimums, lengths, and average steepness over the interval	Compare… the graphs of the functions

EXERCISES

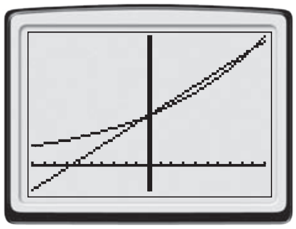
GUIDED PRACTICE

1. Slope: For Fay, use (0, 425) and (1, 375). $\frac{425 - 375}{1 - 0} = 50$. For Kara, use (0, 500) and (1, 425). $\frac{500 - 425}{1 - 0} = 75$. Kara is withdrawing at a higher rate ($75/wk) than Fay ($50/wk); y-int.: The y-intercept for Kara is (0, 500) and for Fay is (0, 425). Kara started with more money ($500) than Fay ($425).

2. Average rate of change of Bacteria X: use (0, 5) and (4, 405). $\frac{405 - 5}{4 - 0} = 100$; X: increased 100 bacteria/h; Average rate of change of Bacteria Y:
use (0, 10) and (4, 160). $\frac{160 - 10}{4 - 0} = 37.5$; Y: increased about 37.5 bacteria/h.

3. Design A's model:
max ht.: $x = -\frac{b}{2a} = -\frac{8}{2(-2)} = \frac{8}{4} = 2$;
$y = -2(2)^2 + 8(2) = 8$ ft; max = 8;
(x–intercepts)
$-2x^2 + 8x = 0$
$-2x(x - 4) = 0$
$x = 0$ or $x = 4$
x-int. = 0, 4;
avg rate of chg. over [0, 2]
At $x = 0$, $y = -2(0)^2 + 8(0) = 0$
At $x = 2$, $y = -2(2)^2 + 8(2) = 8$
Use (0, 0) and (2, 8) = $\frac{8 - 0}{2 - 0} = \frac{8}{2} = 4$;
Design B's model:
B: x-int. = 0, 6; max = 9;
avg rate of chg. over [0, 2]
Use (0, 0) and (2, 8) = $\frac{8 - 0}{2 - 0} = \frac{8}{2} = 4$;
B is taller and wider. Both are equally steep over [0, 2].

4. Plan A is modeled by the equation $y = 30x + 200$. Plan B is modeled by the equation $y = 200(1.10)^x$.



Plan A will result in more bicycles at first, but plan B surpasses plan A by the end of the 8th yr. After that, B exceeds A by ever-increasing amounts each yr.

PRACTICE AND PROBLEM SOLVING

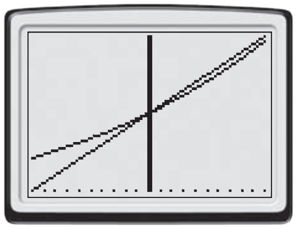
5. Slope: For Darius, the slope is 2.2. For Kevin, the slope is (using (0, 1.5) and (1, 3.5)) $\frac{3.5 - 1.5}{1 - 0} = 2$. Darius is hiking faster (2.2 mi/h) than Kevin (2 mi/h); y-int.: Kevin started farther away from camp (1.5 mi) than Darius (1 mi).

6. Average rate of change of City A: use (0, 0) and (40, 252). $\frac{252 - 0}{40 - 0} = 6.3$ people/mi^2;
Average rate of change of City B: use (0, 0) and (40, 210). $\frac{210 - 0}{40 - 0} = 5.25$ people/mi^2

7. Rocket A's model:
max ht.: $x = -\frac{b}{2a} = -\frac{80}{2(-16)} = \frac{80}{32} = 2.5$;
$y = -16(2.5)^2 + 80(2.5) = 100$ ft;
(x-intercepts)
$-16x^2 + 80x = 0$

$-x(16x - 80) = 0$
$x = 0$ or $x = 5$
x-int. = 0, 5;
avg. rate of chg. over [0, 2]:
At $x = 0$, $y = -16(0)^2 + 80(0) = 0$
At $x = 2$, $y = -16(2)^2 + 80(2) = 96$
Use (0, 0) and (2, 96) = $\frac{96 - 0}{2 - 0} = \frac{96}{2} = 48$;
Rocket B's model: x-int. = 0, 4; max = 64;
avg. rate of chg. over [0, 2]:
Use (0, 0) and (2, 64) = $\frac{64 - 0}{2 - 0} = \frac{64}{2} = 32$.
A is faster, will go higher, and covers more horiz. dist. than B.

8. Proposal A is modeled by the equation $y = 5x + 75$. Proposal B is modeled by the equation $y = 75(1.05)^x$.



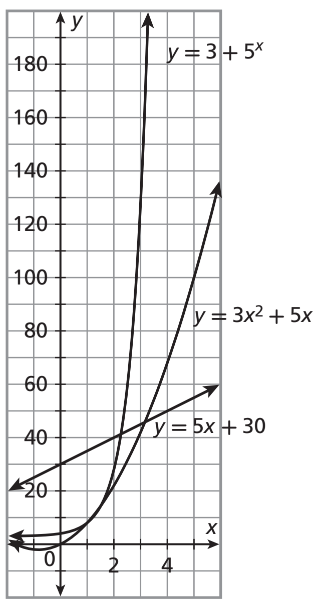
Proposal A will result in more boats over the first several years, but B surpasses A by the end of the 12th yr. After that, B exceeds A by ever-increasing amounts each yr.

9.

$y = 5x + 30$	
x	**y**
0	30
1	35
2	40
3	45
4	50
rate of chg. over [0, 4] = 5	

$y = 3 + 5^x$	
x	**y**
0	4
1	8
2	28
3	128
4	628
avg. rate of chg. over [0, 4] = 156	

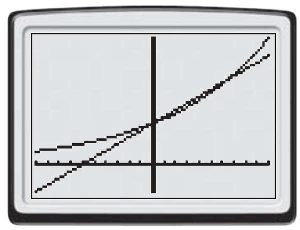
$y = 3x^2 + 5x$	
x	**y**
0	0
1	8
2	22
3	42
4	68
avg. rate of chg. over [0, 4] = 17	



- $y = 5x + 30$ is linear. It has the greatest y-value/highest graph at $x = 0$, but because it has the least rate of change over [0, 4], it has the shallowest graph and the least y-value/lowest graph at $x = 4$.
- $y = 3 + 5^x$ is exponential, has the greatest rate of change over [0, 4], increases the most quickly, and has the steepest graph. In fact, because its rate of change is much greater than the other two functions, it increases much more quickly. Its graph is above the other two graphs by $x \approx 2.5$; after this point, its graph continues to be above the other two by ever-increasing amounts.
- $y = 3x^2 + 5x$ is quadratic. On [0, 4], it starts out less than $y = 5x + 30$ with its graph below the graph of $y = 5x + 30$, but because $y = 3x^2 + 5x$ has the greater rate of change, it eventually exceeds $y = 5x + 30$ (at $x \approx 3.5$); from this point on, $y = 3x^2 + 5x$ is greater than $y = 5x + 30$, the graph of $y = 3x^2 + 5x$ is steeper, and it is higher than the graph of $y = 5x + 30$ by ever-increasing amounts. However, $y = 3x^2 + 5x$ does not grow as quickly as $y = 3 + 5^x$. The graph of $y = 3x^2 + 5x$ is below the graph of $y = 3 + 5^x$, and it is not as steep.

10. Average rate of change: Use (0, −100) and (4, 380) = $\frac{380 - (-100)}{4 - 0} = \frac{480}{4} = 120$
maximum: $5

11a. Plan A is modeled by the equation $y = 120(1.12)^x$. Plan B is modeled by the equation $y = 20x + 120$.



Plan B results in more students in the first few years, but plan A exceeds plan B by year 7, after which A continues to exceed B by ever-increasing amounts each year.

b. B; B will double enrollment in 6 yrs, while A will take slightly longer.

c. A; A will triple enrollment in 9–10 yrs, while B will take 12–13 yrs.

12. Lin.: graph is a line; const. rate of chg; 0 or 1 x-int.; no min or max; const. 1st differences. Quad.: graph is a parabola; var. rate of chg; 0, 1, or 2 x-int.; has min or max at vertex; const. 2nd differences. Exp.: var. rate of chg; no x-int.; no min or max; const. ratios.

STANDARDIZED TEST PREP

13. Plan A is modeled by the equation $y = 1000x + 2000$. Plan B is modeled by the equation $y = 2000(1.20)^x$. After 10 years:
Plan A: $y = 1000(10) + 2000 = 12{,}000$
Plan B: $y = 2000(1.20)^{10} = 12{,}383$

Difference = 12,383 − 12,000 = 383.
Choice A is correct.

14. For $t = 2$, $h = 64(2) - 16(2)^2 = 64$. Choice J is correct.

CHALLENGE AND EXTEND

15a. For [0, 4]:
Function A ($y = -x^2 + 12x$): Use (0, 0) and (4, 32).
$\frac{32 - 0}{4 - 0} = 8$
Function B: Use (0, 11) and (4, −4).
$\frac{-4 - 11}{4 - 0} = \frac{-15}{4} = -3.75$
For [0, 6]:
Function A ($y = -x^2 + 12x$): Use (0, 0) and (6, 36).
$\frac{36 - 0}{6 - 0} = 6$
Function B: Use (0, 11) and (6, −52).
$\frac{-52 - 11}{6 - 0} = \frac{-63}{6} = -10.5$
For [4, 6]:
Function A ($y = -x^2 + 12x$): Use (4, 32) and (6, 36).
$\frac{36 - 32}{6 - 4} = 2$
Function B: Use (4, −4) and (6, −52).
$\frac{-52 - (-4)}{6 - 4} = \frac{-48}{2} = -24$
For [7, 10]:
Function A ($y = -x^2 + 12x$): Use (7, 35) and (10, 20). $\frac{20 - 35}{10 - 7} = \frac{-15}{3} = -5$
Function B: Use (7, −116) and (10, −1012).
$\frac{-1012 - (-116)}{10 - 7} = \frac{-896}{3} = -298.7$
For [8, 12]:
Function A ($y = -x^2 + 12x$): Use (8, 32) and (12, 0).
$\frac{0 - 32}{12 - 8} = \frac{-32}{4} = -8$
Function B: Use (8, −244) and (12, −4084).
$\frac{-4084 - (-244)}{12 - 8} = \frac{-3840}{4} = -960$

	A	B
[0, 4]	8	−3.75
[0, 6]	6	−10.5
[4, 6]	2	−24
[7, 10]	−5	−298.7
[8, 12]	−8	−960

b. A: rate of chg is pos. and decreasing until (6, 36), when it becomes neg. and decreasing. B has ever−decreasing neg. rate of chg.

c. It is halved.

READY TO GO ON? Section B Quiz

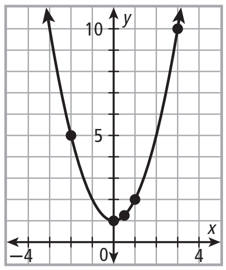
1. $y = a(1 + r)^x$
$= 30{,}000(1.03)^x;$
After 10 years, $y = \$40{,}317.49$

2. $y = a(1 + \frac{r}{n})^{nx}$
$= 2000(1.00375)^{12x;}$
After 3 years, $y = \$2288.50$

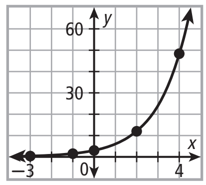
3. $y = a(1 - r)^x$
$= 1200(0.8)^x$
After 4 years, $y = \$491.52$

4. $A = P(0.5)^t$
$A = 100(0.5)^{\frac{300}{30}}$
$A = 100(0.5)^{10}$
$A = 0.098$ mg

5. quadratic



6. exponential



7. linear; for every constant change of +1 in the x-values, there is a constant change of +1 in the y-values.

8. exponential: for every constant change of +1 in the x-values, there is a common ratio of $\frac{1}{2}$ in the y-values.

9. The value of the stamp is increasing by 20% each year; $y = 5(1.2)^x$; \$37.15

10. A: x-int. = 0,55; max = 121; avg. rate of chg. over [0, 2] = 64; B: x-int. = 0,6; max = 144; avg. rate of chg. over [0, 2] = 72. B is faster, will go higher, and stays in the air longer than A.

STUDY GUIDE: REVIEW

EXERCISES

1. square-root function

2. exponential decay

3. common ratio

4. exponential function

9-1 GEOMETRIC SEQUENCES

5. $\frac{3}{1} = 3;\ \frac{9}{3} = 3;\ \frac{27}{9} = 3$
The next three terms are: 27 • 3 = 81
81 • 3 = 243
243 • 3 = 729