

44. Possible answer: The following table shows how much money you could earn with each plan.

Year	Salary Plan A	Salary Plan B
0	\$0	\$10,000
1	\$20,000	\$20,000
2	\$40,000	\$40,000
3	\$60,000	\$80,000

Choose plan B because plan A doesn't pay anything for the first year and because after 3 years, plan B pays more money.

45. C; the other graphs do not increase exponentially.

46. G;  $f(4) = 15(1.4)^2 = 29.4$

47. D;  $a_1 = 5$ ,  $r = 5$ , hence  $a_n = 5(5)^{n-1} = 5^n$

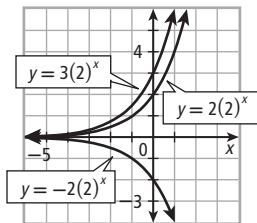
### CHALLENGE AND EXTEND

48.  $4^x = 64$   
 $4^x = 4^3$   
 $x = 3$

49.  $\left(\frac{1}{3}\right)^x = \frac{1}{27}$   
 $3^{-x} = 3^{-3}$   
 $-x = -3$   
 $x = 3$

50.  $2^x = \frac{1}{16}$   
 $2^x = \frac{1}{2^4}$   
 $2^x = 2^{-4}$   
 $x = -4$

51. The value of  $a$  is the  $y$ -intercept.



### READY TO GO ON? Section A Quiz

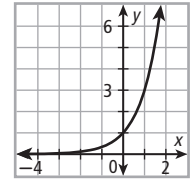
- $\frac{6}{3} = 2$ ;  $\frac{12}{6} = 2$ ;  $\frac{24}{12} = 2$ ; the common ratio is 2  
 next 3 terms:  $24(2) = 48$ ,  $48(2) = 96$ , and  $96(2) = 192$
- $\frac{2}{-1} = -2$ ;  $\frac{-4}{2} = -2$ ;  $\frac{8}{-4} = -2$ ;  
 the common ratio is  $-2$   
 next 3 terms:  $8(-2) = -16$ ,  $(-16)(-2) = 32$ , and  $32(-2) = -64$
- $\frac{-1200}{-2400} = \frac{1}{2}$ ;  $\frac{-600}{-1200} = \frac{1}{2}$ ;  $\frac{-300}{-600} = \frac{1}{2}$   
 next 3 terms:  $-300\left(\frac{1}{2}\right) = -150$ ,  $-150\left(\frac{1}{2}\right) = -75$ ,  
 and  $-75\left(\frac{1}{2}\right) = -37.5$

4.  $a_n = a_1 r^{n-1}$   
 $a_8 = (2)(3)^{8-1}$   
 $a_8 = 4374$

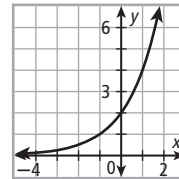
5.  $a_1 = 1000$ ,  $r = \frac{4}{5}$   
 $a_n = a_1 r^{n-1}$   
 $a_7 = 1000\left(\frac{4}{5}\right)^{7-1}$   
 $a_7 = 262.144 \text{ cm}$

6.  $f(x) = 3(1.1)^x$   
 $f(4) = 3(1.1)^4$   
 $f(4) = 4.39 \text{ in}$

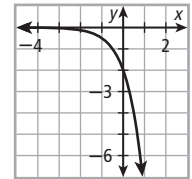
7.  $y = 3^x$



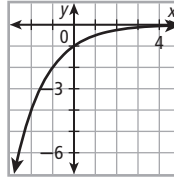
8.  $y = 2(2)^x$



9.  $y = -2(4)^x$



10.  $y = -(0.5)^x$



11.  $f(x) = 40(0.8)^x$   
 $2 = 40(0.8)^x$   
 $x \approx 14$ ;  
 after about 14 h

## 9-3 EXPONENTIAL GROWTH AND DECAY

### CHECK IT OUT!

1.  $y = a(1 + r)^t$   
 $= 1200(1.08)^t$ ;  
 In 2006,  $y = 1200(1.08)^6 = \$1904.25$

2a.  $A = P\left(1 + \frac{r}{n}\right)^{nt}$   
 $= 1200\left(1 + \frac{0.035}{4}\right)^{4t}$   
 $= 1200(1.00875)^{4t}$   
 After 4 years,  $A = 1200(1.00875)^{16} = \$1379.49$

b.  $A = P\left(1 + \frac{r}{n}\right)^{nt}$   
 $= 4000\left(1 + \frac{0.03}{12}\right)^{12t}$   
 $= 4000(1.0025)^{12t}$   
 After 8 years,  $A = 4000(1.0025)^{96} = \$5083.47$

3.  $y = a(1 - r)^t$   
 $= 48,000(1 - 0.03)^t$   
 $= 48,000(0.97)^t$   
 After 7 years,  $y = 48,000(0.97)^7 = 38,783$

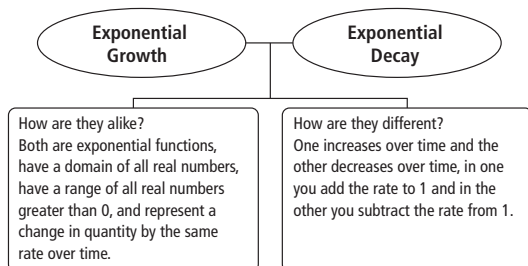
4a.  $t = \frac{180 \text{ years}}{30 \text{ years}}$   
 $= 6$   
 $A = P(0.5)^t$   
 $= 100(0.5)^6$   
 $= 1.5625 \text{ mg}$

$$\begin{aligned} \text{b. } t &= \frac{5 \text{ weeks}}{5 \text{ days}} \\ &= 7 \\ A &= P(0.5)^t \\ &= 100(0.5)^7 \\ &= 0.78125 \text{ g} \end{aligned}$$

## THINK AND DISCUSS

- Possible answers: interest earned on an investment, population growth or decline, radioactive decay
- increasing; by 2% per year
- An exponential growth function has the form  $y = a(1 + r)^t$ . The base  $(1 + r)$  corresponds to the base  $b$ . The exponent  $t$  corresponds to the exponent  $x$ . An exponential decay function has the form  $y = a(1 - r)^t$ . The base  $(1 - r)$  corresponds to the base  $b$ . The exponent  $t$  corresponds to the exponent  $x$ .

4.



## EXERCISES

### GUIDED PRACTICE

- exponential growth, since  $2 > 1$ .

$$\begin{aligned} 2. \ y &= a(1 + r)^t \\ &= 12,000(1 + 0.06)^t \\ &= 12,000(1.06)^t \\ \text{After 4 years, } y &= 12,000(1.06)^4 = \$15,149.72 \end{aligned}$$

$$\begin{aligned} 3. \ y &= a(1 + r)^t \\ &= 300(1 + 0.08)^t \\ &= 300(1.08)^t \\ \text{After 5 years, } y &= 300(1.08)^5 = 441 \end{aligned}$$

$$\begin{aligned} 4. \ A &= P\left(1 + \frac{r}{n}\right)^{nt} \\ &= 1500\left(1 + \frac{0.035}{1}\right)^t \\ &= 1500(1.035)^t \\ \text{After 4 years, } A &= 1500(1.035)^4 = \$1721.28 \end{aligned}$$

$$\begin{aligned} 5. \ A &= P\left(1 + \frac{r}{n}\right)^{nt} \\ &= 4200\left(1 + \frac{0.028}{4}\right)^{4t} \\ &= 4200(1.007)^{4t} \\ \text{After 6 years, } A &= 4200(1.007)^{24} = \$4965.43 \end{aligned}$$

$$\begin{aligned} 6. \ y &= a(1 - r)^t \\ &= 18,000(1 - 0.12)^t \\ &= 18,000(0.88)^t \\ \text{After 10 years, } y &= 18,000(0.88)^{10} = \$5013.02 \end{aligned}$$

$$\begin{aligned} 7. \ y &= a(1 - r)^t \\ &= 10(1 - 0.16)^t \\ &= 10(0.84)^t \\ \text{After 4 hours, } y &= 10(0.84)^4 = 4.98 \text{ mg} \end{aligned}$$

8. $t = \frac{1 \text{ hr}}{20 \text{ min}}$ $= 3$ $A = P(0.5)^t$ $= 30(0.5)^3$ $= 3.75 \text{ g}$	9. $t = \frac{156 \text{ days}}{52 \text{ days}}$ $= 3$ $A = P(0.5)^t$ $= 44(0.5)^3$ $= 5.5 \text{ g}$
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### PRACTICE AND PROBLEM SOLVING

$$\begin{aligned} 10. \ y &= a(1 + r)^t \\ &= 149,000(1.06)^t \\ \text{After 7 years, } y &= 149,000(1.06)^7 = \$224,040.91 \end{aligned}$$

$$\begin{aligned} 11. \ y &= a(1 + r)^t \\ &= 1600(1 + 0.03)^t \\ &= 1600(1.03)^t \\ \text{After 10 years, } y &= 1600(1.03)^{10} = 2150 \end{aligned}$$

$$\begin{aligned} 12. \ A &= P(1 + r)^t \\ &= 700(1 + 0.012)^t \\ &= 700(1.012)^t \\ \text{After 8 years, } A &= 700(1.012)^8 = \$770.09 \end{aligned}$$

$$\begin{aligned} 13. \ y &= P(1 + r)^t \\ &= 30(1 + 0.078)^t \\ &= 30(1.078)^t \\ \text{After 6 years, } y &= 30(1.078)^6 = 47 \text{ members} \end{aligned}$$

$$\begin{aligned} 14. \ A &= P\left(1 + \frac{r}{n}\right)^{nt} \\ &= 28,000(1 + 0.04)^t \\ &= 28,000(1.04)^t \\ \text{After 5 years, } A &= 28,000(1.04)^5 = \$34,066.28 \end{aligned}$$

$$\begin{aligned} 15. \ A &= P\left(1 + \frac{r}{n}\right)^{nt} \\ &= 7000\left(1 + \frac{0.03}{4}\right)^{4t} \\ &= 7000(1.0075)^{4t} \\ \text{After 10 years, } A &= 7000(1.0075)^{40} = \$9438.44 \end{aligned}$$

$$\begin{aligned} 16. \ A &= P\left(1 + \frac{r}{n}\right)^{nt} \\ &= 3500\left(1 + \frac{0.018}{12}\right)^{12t} \\ &= 3500(1.0015)^{12t} \\ \text{After 4 years, } A &= 3500(1.0015)^{48} = \$3761.09 \end{aligned}$$

$$\begin{aligned} 17. \ A &= P\left(1 + \frac{r}{n}\right)^{nt} \\ &= 12,000(1 + 0.026)^t \\ &= 12,000(1.026)^t \\ \text{After 15 years, } A &= 12,000(1.026)^{15} = \$17,635.66 \end{aligned}$$

$$\begin{aligned} 18. \ y &= a(1 - r)^t \\ &= 18,000(1 - 0.02)^t \\ &= 18,000(0.98)^t \\ \text{After 6 years, } y &= 18,000(0.98)^6 = 15,945 \end{aligned}$$

$$\begin{aligned}
 19. \quad y &= a(1 - r)^t \\
 &= 58(1 - 0.1)^t \\
 &= 58(0.9)^t \\
 \text{After 8 years, } y &= 58(0.9)^8 = \$24.97
 \end{aligned}$$

$$\begin{aligned}
 20. \quad t &= \frac{6 \text{ days}}{36 \text{ hours}} \\
 &= \frac{144 \text{ hours}}{36 \text{ hours}} \\
 &= 4 \\
 A &= P(0.5)^t \\
 &= 80(0.5)^4 \\
 &= 5 \text{ g}
 \end{aligned}$$

21. growth; 61%, since  $1 + r = 1.61$

22. decay; 90.2%, since  $1 - r = 0.098$

23. decay;  $33\frac{1}{3}\%$ , since  $1 - r = \frac{2}{3}$

24. growth; 50%, since  $1 + r = \frac{3}{2}$

25. growth; 10%, since  $1 + r = 1.1$

26. decay; 20%, since  $1 - r = 0.8$

27. growth; 25%, since  $1 + r = \frac{5}{4}$

28. decay; 50%, since  $1 - r = \frac{1}{2}$

$$\begin{aligned}
 29. \quad y &= a(1 + r)^t \\
 &= 58,000,000(1.001)^t \\
 \text{After 3 years, } y &= 58,000,000(1.001)^3 \\
 &= 58,174,174
 \end{aligned}$$

$$\begin{aligned}
 30. \quad y &= a(1 + r)^t \\
 &= 32,000(1.07)^t \\
 \text{After 5 years, } y &= 32,000(1.07)^5 = \$44,881.66
 \end{aligned}$$

$$\begin{aligned}
 31. \quad y &= a(1 - r)^t \\
 &= 8200(1 - 0.02)^t \\
 &= 8200(0.98)^t \\
 \text{After 7 years, } y &= 8200(0.98)^7 = \$7118.63
 \end{aligned}$$

$$\begin{aligned}
 32. \quad y &= a(1 - r)^t \\
 &= 25,000(1 - 0.15)^t \\
 &= 25,000(0.85)^t \\
 \text{After 6 years, } y &= 25,000(0.85)^6 = \$9428.74
 \end{aligned}$$

$$\begin{aligned}
 33. \quad y &= a(1 + r)^t \\
 &= 970(1 + 0.012)^t \\
 &= 970(1.012)^t \\
 \text{After 5 years, } y &= 970(1.012)^5 = 1030
 \end{aligned}$$

$$\begin{aligned}
 34. \quad t &= \frac{3500 \text{ years}}{5700 \text{ years}} \\
 &= \frac{35}{57} \\
 A &= P(0.5)^t \\
 &= 15(0.5)^{\frac{35}{57}} \\
 &\approx 9.80 \text{ g}
 \end{aligned}$$

35. B; possible answer: student B did not subtract the rate from 1.

36. No; possible answer; there is no value for  $t$  that would make  $(0.84)^t$  equal 0.

$$\begin{aligned}
 37. \quad y &= a(1 + r)^t \\
 600 &= 300(1 + 0.04)^t \\
 2 &= 1.04^t \\
 t &\approx 18 \text{ years}
 \end{aligned}$$

$$\begin{aligned}
 38a. \quad y &= a(1 + r)^t \\
 &= 20,000(1.09)^t
 \end{aligned}$$

b. In 2008,  $t = 6$ , hence  
 $y = 20,000(1.09)^6 = \$33,542$

c. 2011

Year	Tuition (\$)
2002	20,000
2003	21,800
2004	23,762
2005	25,900.58
2006	28,231.63
2007	30,772.48
2008	33,542.00
2009	36,560.78
2010	39,851.25
2011	43,437.87

39. In 10 years:

$$A: 600(1.05)^{10} = \$977.34$$

$$\begin{aligned}
 B: 500\left(1 + \frac{0.06}{4}\right)^{40} &= 500(1.015)^{40} \\
 &= \$907.01
 \end{aligned}$$

A will have a larger balance.

In 20 years:

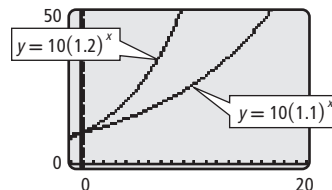
$$A: 600(1.05)^{20} = \$1591.98$$

$$B: 500(1.015)^{80} = \$1645.33$$

B will have a larger balance.

40. 50 h; 15h

41. The graph when  $r$  is 20% rises faster than when  $r$  is 10%. The greater the value of  $r$ , the faster the graph will rise.



42. Possible answer: \$400 is invested at a rate of 8% compounded annually.

43. Possible answer: The population is 800 and decreasing at a rate of 4% per year.

44. No; possible answer: the sample doubles every minute, so the container is half full 1 minute before it is full. This would be after 5 min.

$$\begin{aligned}
 45. \quad D; \quad y &= a(1 - r)^t \\
 a &= 500, \quad 1 - r = 1 - 0.01 = 0.99
 \end{aligned}$$

46. G;  $a = -5$ , so as the absolute value of  $y$  decreases,  $y$  is actually increasing.

47. D;  $865(1.05)^3 = \$1001.35$

48a.  $y = a(1 + r)^t$   
 $= 1000(1 + 0.05)^t$   
 $= 1000(1.05)^t$

b.  $1300 = 1000(1.05)^t$   
 $t \approx 5$ ; about 2005

### CHALLENGE AND EXTEND

49. about 20 years

50.  $y = a(1 + r)^t$   
 $1000 = 500(1.04)^t$   
 $t \approx 18$  yr  
for  $r = 0.08$   
 $1000 = 500(1.08)^t$   
 $t \approx 9$  yr

51.  $A = P(0.5)^t$   
 $10 = 80(0.5)^t$   
 $t = 3$   
So, half-life =  $\frac{300}{t} = 100$  min or 1 h 40 min

52.  $A = P(0.5)^t$   
 $15 = P(0.5)^{\frac{6}{2}}$   
 $P = 120$  g

53.  $A = P\left(1 + \frac{r}{n}\right)^{nt}$   
 $250,000 = P(1 + 0.013)^{(4 \cdot 8)}$   
 $P = \$225,344$

54. Month	Balance (\$)	Monthly Payment (\$)	Remaining Balance (\$)	1.5% Finance Charge (\$)	New Balance (\$)
1	200	30	170	2.55	172.55
2	172.55	30	142.55	2.14	144.69
3	144.69	30	114.69	1.72	116.41
4	116.41	30	86.41	1.30	87.71
5	87.71	30	57.71	0.87	58.58
6	58.58	30	28.58	0.43	29.01
7	29.01	29.01	0	0	0

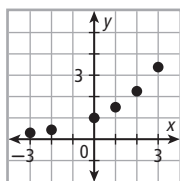
b. Table shows balance is paid off in 7 months.

c.  $(6(30) + 29.01) - 200 = 9.01$

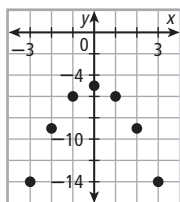
## 9-4 LINEAR, QUADRATIC AND EXPONENTIAL MODELS

### CHECK IT OUT!

1a. exponential



b. quadratic



2. Quadratic; for every constant change in the  $x$ -values of  $+1$ , there is a constant second difference of  $-6$  in the  $y$ -values.

3. The oven temperature decreases by  $50^\circ\text{F}$  every 10 minutes;  $y = -5x + 375$ ;  $75^\circ\text{F}$

### THINK AND DISCUSS

1. No; most real-world data probably will not fit exactly into one of these patterns.
2. No; this is just a prediction based on the assumption that the observed trends will continue, which they may or may not do.

3.

Modeling Data

Linear: Points lie on a line; for a constant change in  $x$ , the first differences are constant.

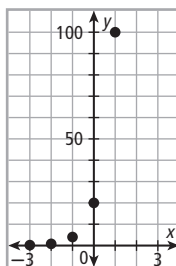
Exponential: Smooth curve that extends infinitely either up to the right, up to the left, down to the right, or down to the left; for a constant change in  $x$ , there is a constant ratio.

Quadratic: Points lie on a parabola and are symmetric with a vertical line through the vertex; for a constant change in  $x$ , the second differences are constant.

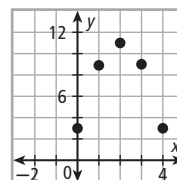
### EXERCISES

#### GUIDED PRACTICE

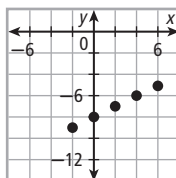
1. exponential



2. quadratic



3. linear



4. Quadratic; for every constant change of  $+1$  in the  $x$ -values, there is a constant second difference of  $-1$  in the  $y$ -values.

5. Exponential; for every constant change of  $+1$  in the  $x$ -values, there is a constant ratio of 2.

6. Linear; for every constant change of  $+1$  in the  $x$ -values, there is a constant change of  $+2$  in the  $y$ -values.

7. Grapes cost  $\$1.79/\text{lb}$ ;  $y = 1.79x$ ;  $\$10.74$