CHAPTER 9 Exponential and Radical Functions
Solutions Key

ARE YOU READY

1. B; like terms: terms that contain the same variable raised to the same power

2. F; square root: one of two equal factors of a number

3. C; domain: the set of first elemtns of a relation

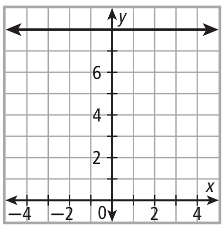
4. E; perfect square: a number whose positive square root is a whole number

5. D; exponent: a number that tells how many times a base is used as a factor

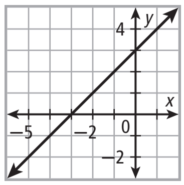
6. 16 **7.** 1 **8.** 63 **9.** 375

10. 243 **11.** −28 **12.** 320 **13.** 147

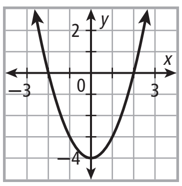
14. $y = 8$



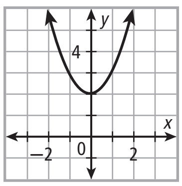
15. $y = x + 3$



16. $y = x^2 - 4$



17. $y = x^2 + 2$



18. 0.5 **19.** 0.25 **20.** 0.152 **21.** 2.0

22. 0.019 **23.** 0.003 **24.** 0.001 **25.** 0.0104

26. 6; $6 \cdot 6 = 36$ **27.** 9; $9 \cdot 9 = 81$

28. 5; $5 \cdot 5 = 25$ **29.** 8; $8 \cdot 8 = 64$

30. $h^2 = 3^2 + 4^2$
$h^2 = 25$
$h = 5$ cm

31. $h^2 = 12^2 + 5^2$
$h^2 = 169$
$h = 13$ in.

32. $h^2 = 6^2 + 8^2$
$h^2 = 100$
$h = 10$ ft

33. $5(2m - 3) = 5 \cdot 2m - 5 \cdot 3$
$= 10m - 15$

34. $3x(8x + 9) = 3x \cdot 8x + 3x \cdot 9$
$= 24x^2 + 27x$

35. $2t(3t - 1) = 2t \cdot 3t - 2t \cdot 1$
$= 6t^2 - 2t$

36. $4r(4r - 5) = 4r \cdot 4r - 4r \cdot 5$
$= 16r^2 - 20r$

9-1 GEOMETRIC SEQUENCES

CHECK IT OUT!

1a. 80, −160, 320; $(-10) \div 5 = -2$,
$20 \div (-10) = -2$, $(-40) \div 20 = -2$
So, the common ratio is −2.
$(-40) \cdot (-2) = 80$, $80 \cdot (-2) = -160$,
and $(-160) \cdot (-2) = 320$

b. 216, 162, 121.5; $384 \div 512 = \frac{3}{4}$, $288 \div 384 = \frac{3}{4}$,
So, the common ratio is $\frac{3}{4}$.
$288 \cdot \frac{3}{4} = 216$, $216 \cdot \frac{3}{4} = 162$,
and $162 \cdot \frac{3}{4} = 121.5$

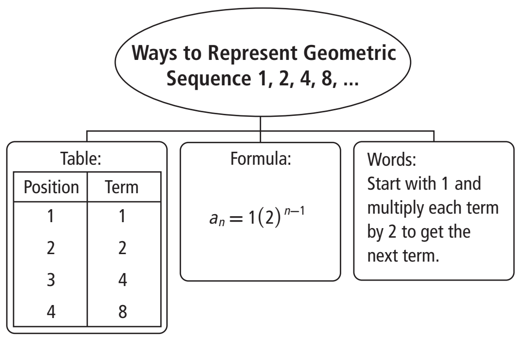
2. $a_n = a_1 r^{n-1}$
$a_8 = 1000\left(\frac{1}{2}\right)^7$
$a_8 = 7.8125$

3. $a_n = a_1 r^{n-1}$
$a_{10} = 10{,}000\left(\frac{4}{5}\right)^9$
$a_{10} = 1342.18$;
$1342.18

THINK AND DISCUSS

1. Possible answer: Divide each term after the first by the preceding term. If the quotients are all the same, the sequence is geometric.

2. Possible answer:



EXERCISES

GUIDED PRACTICE

1. common ratio: the value that each term is multiplied by to get the next term.

2. 32, 64, 128; $4 \div 2 = 2$, $8 \div 4 = 2$, $16 \div 8 = 2$
So, the common ratio is 2.
Then, $16 \cdot 2 = 32$, $32 \cdot 2 = 64$,
and $64 \cdot 2 = 128$

3. 25, 12.5, 6.25; $200 \div 400 = \frac{1}{2}$,
$100 \div 200 = \frac{1}{2}$, $50 \div 100 = \frac{1}{2}$
So, the common ratio is $\frac{1}{2}$.
Then, $50 \cdot \frac{1}{2} = 25$, $25 \cdot \frac{1}{2} = 12.5$,
and $12{,}5 \cdot \frac{1}{2} = 6.25$

4. 324, -972, 2916; $(-12) \div 4 = -3$,
$36 \div (-12) = -3$, $(-108) \div 36 = -3$
So, the common ratio is -3.
Then, $(-108) \cdot (-3) = 324$, $324 \cdot (-3) = -972$,
and $(-972) \cdot (-3) = 2916$

5. $a_n = a_1 r^{n-1}$
$a_{10} = 1 \cdot 10^{10-1}$
$a_{10} = 1{,}000{,}000{,}000$

6. $a_n = a_1 r^{n-1}$
$a_{11} = 3 \cdot 2^{11-1}$
$a_{11} = 3072$

7. $\frac{32}{64} = \frac{1}{2}$; $\frac{16}{32} = \frac{1}{2}$
$a_n = a_1 r^{n-1}$
$a_5 = 64 \cdot \left(\frac{1}{2}\right)^4$
$a_5 = 4$

PRACTICE AND PROBLEM SOLVING

8. -1250, 6250, $-31{,}250$; $\frac{10}{-2} = -5$;
$\frac{-50}{10} = -5$; $\frac{250}{-50} = -5$
So, the common ratio is -5.
Then, $250 \cdot (-5) = -1250$, $-1250 \cdot (-5) = 6250$,
and 6250, $\cdot (-5) = -31\ 250$

9. 162, 243, 364.5; $\frac{48}{32} = \frac{3}{2}$; $\frac{72}{48} = \frac{3}{2}$; $\frac{108}{72} = \frac{3}{2}$
So, the common ratio is $\frac{3}{2}$.
Then, $108\left(\frac{3}{2}\right) = 162$, $162\left(\frac{3}{2}\right) = 243$,
and $243\left(\frac{3}{2}\right) = 364.5$

10. 256, 204.8, 163.84; $\frac{500}{625} = \frac{4}{5}$; $\frac{400}{500} = \frac{4}{5}$; $\frac{320}{400} = \frac{4}{5}$
So, the common ratio is $\frac{4}{5}$.
Then, $320\left(\frac{4}{5}\right) = 256$, $256\left(\frac{4}{5}\right) = 204.8$,
and $204.8\left(\frac{4}{5}\right) = 163.84$

11. 2058, 14 406, 100 842; $\frac{42}{6} = 7$; $\frac{294}{42} = 7$
So, the common ratio is 7.
Then, $294 \cdot 7 = 2058$, $2058 \cdot 7 = 14,406$
and $14,406 \cdot 7 = 100,842$

12. 96, -192, 384; $-\frac{12}{6} = -2$; $\frac{24}{-12} = -2$; $\frac{-48}{24} = -2$
So, the common ratio is -2.
Then, $-48\,(-2) = 96$, $96\,(-2) = -192$, and
$-192\,(-2) = 384$

13. $\frac{5}{32}$, $\frac{5}{128}$, $\frac{5}{512}$; $\frac{10}{40} = \frac{1}{4}$; $\frac{5}{2} \div 10 = \frac{1}{4}$; $\frac{5}{8} \div \frac{5}{2} = \frac{1}{4}$
So, the common ratio is $\frac{1}{4}$.
Then, $\left(\frac{5}{8}\right)\left(\frac{1}{4}\right) = \frac{5}{32}$, $\left(\frac{5}{32}\right)\left(\frac{1}{4}\right) = \frac{5}{128}$, and
$\left(\frac{5}{128}\right)\left(\frac{1}{4}\right) = \frac{5}{512}$

14. $a_n = a_1 r^{n-1}$
$a_5 = 18 \cdot (3.5)^{5-1}$
$a_5 = 2701.125$

15. $\frac{100}{1000} = \frac{1}{10}$; $\frac{10}{100} = \frac{1}{10}$; $\frac{1}{10} = \frac{1}{10}$
$a_n = a_1 r^{n-1}$
$a_{14} = 1000 \cdot 0.1^{14-1}$
$a_{14} = 0.0000000001$ or $a_{14} = 1 \times 10^{-10}$

16. 83.9 m; $\frac{320}{400} = \frac{4}{5}$; $\frac{256}{320} = \frac{4}{5}$
$a_n = a_1 r^{n-1}$
$a_8 = 400\left(\frac{4}{5}\right)^{8-1}$
$a_8 = 83.9$

17. 20, 40, **80**, **160**; $\frac{40}{20} = 2$, so the common ratio is 2;
$40 \cdot 2 = 80$ and $80 \cdot 2 = 160$

18. **2**, 6, 18, **54**; $\frac{18}{6} = 3$, so the common ratio is 3;
$\frac{6}{3} = 2$ and $18 \cdot 3 = 54$

19. 9, 3, 1, $\mathbf{\frac{1}{3}}$; $\frac{3}{9} = \frac{1}{3}$; $\frac{1}{3} = \frac{1}{3}$
So the common ratio is $\frac{1}{3}$; $1 \cdot \frac{1}{3} = \frac{1}{3}$

20. 3, 12, **48**, 192, **768**; $\frac{12}{3} = 4$, so the common ratio
is 4; $12 \cdot 4 = 48$ and $192 \cdot 4 = 768$

21. 7, 1, $\mathbf{\frac{1}{7}}$, $\mathbf{\frac{1}{49}}$, $\frac{1}{343}$; The common ratio is $\frac{1}{7}$;
$1 \cdot \frac{1}{7} = \frac{1}{7}$ and $\frac{1}{7} \cdot \frac{1}{7} = \frac{1}{49}$

22. **400**, 100, 25, $\mathbf{\frac{25}{4}}$; $\frac{25}{100} = \frac{1}{4}$, so the common ratio
is $\frac{1}{4}$.
Then, $100 \div \frac{1}{4} = 25$ and $25 \cdot \frac{1}{4} = \frac{25}{4}$

23. -3, **6**, -12, 24, **-48**; $\frac{24}{-12} = -2$, so the common
ratio is -2.
Then, $-3 \cdot (-2) = 6$ and $24 \cdot (-2) = -48$

24. $\mathbf{\frac{1}{9}}$, $\mathbf{-\frac{1}{3}}$, 1, -3, 9; $-\frac{3}{1} = -3$; $\frac{9}{-3} = -3$
So the common ratio is -3.
Then, $1 \div -3 = -\frac{1}{3}$ and $-\frac{1}{3} \div -3 = \frac{1}{9}$

25. 1, 17, 289, **4913**; $\frac{17}{1} = 17$; $\frac{289}{17} = 17$
So the common ratio is 17.
Then, $289 \cdot 17 = 4913$

26. $\frac{10}{2} = 5; \frac{50}{10} = 5; \frac{250}{50} = 5$
The common ratio is 5; yes.

27. $\frac{15}{5} = \frac{1}{3}; \frac{5}{3} \div 5 = \frac{1}{3}; \frac{5}{9} \div \frac{5}{3} = \frac{1}{3}$
The common ratio is $\frac{1}{3}$; yes.

28. $\frac{18}{6} = 3; \frac{24}{18} = \frac{4}{3}; \frac{38}{24} = \frac{19}{12}$
There is no common ratio; no.

29. $\frac{3}{9} = \frac{1}{3}; \frac{-1}{3} = -\frac{1}{3}; \frac{-5}{-1} = 5$
There is no common ratio; no.

30. $\frac{21}{7} = 3; \frac{63}{21} = 3; \frac{189}{63} = 3$
The common ratio is 3; yes.

31. $\frac{1}{4} = \frac{1}{4}; \frac{-2}{1} = -2; \frac{-4}{-2} = 2$
There is no common ratio; no.

32a. $\frac{2}{1} = 2; \frac{4}{2} = 2; \frac{8}{4} = 2$
Plan 2 is a geometric sequence with common ratio 2.

b. Possible answer: Plan 1; Under Plan 2, the cost for the 10th week alone is \$512, which is more than the cost for the entire summer under Plan 1.

33a. $a_n = a_1 r^{n-1}$
$a_7 = 0.02 \cdot 2^6$
$a_7 = 1.28$ cm

b. $a_n = a_1 r^{n-1}$
$a_{12} = 0.02 \cdot 2^{11}$
$a_{12} = 40.96$ cm

34. $a_1 = 3$
$a_2 = 3\,(2)^1 = 6$
$a_3 = 3\,(2)^2 = 12$
$a_4 = 3\,(2)^3 = 24$

35. $a_1 = -2$
$a_2 = -2\,(4)^1 = -8$
$a_3 = -2\,(4)^2 = -32$
$a_4 = -2\,(4)^3 = -128$

36. $a_1 = 5$
$a_2 = 5\,(-2)^1 = -10$
$a_3 = 5\,(-2)^2 = 20$
$a_4 = 5\,(-2)^3 = -40$

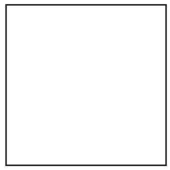
37. $a_1 = 2$
$a_2 = 2\,(2)^1 = 4$
$a_3 = 2\,(2)^2 = 8$
$a_4 = 2\,(2)^3 = 16$

38. $a_1 = 2$
$a_2 = 2\,(5)^1 = 10$
$a_3 = 2\,(5)^2 = 50$
$a_4 = 2\,(5)^3 = 250$

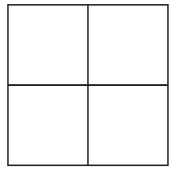
39. $a_1 = 12$
$a_2 = 12\left(\frac{1}{4}\right)^1 = 3$
$a_3 = 12\left(\frac{1}{4}\right)^2 = \frac{3}{4}$
$a_4 = 12\left(\frac{1}{4}\right)^3 = \frac{3}{16}$

40. Each term is multiplied by 2^{n-1}, where n is the term number. For example, begin with the geometric sequence 4, 12, 36, 108. ..., where $r = 3$. If r is doubled to 6, the sequence becomes 4, 24, 144, 864, ...

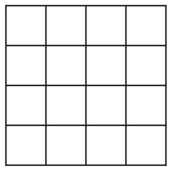
41a. Stage 0



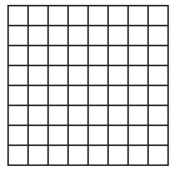
Stage 1:



Stage 2:



Stage 3:



b.

Stage	Squares
0	1
1	4
2	16
3	64

c. $\frac{4}{1} = 4; \frac{16}{4} = 4; \frac{64}{16} = 4$
yes; $r = 4$

d. $r = 4$ and $a_1 = 4$
$a_n = a_1 r^{n-1}$
$a_n = 4\,(4)^{n-1}$
$a_n = 4^n$

42. Divide each term by the preceeding term to find the value of r. Then use the formula $a_n = a_1 r^{n-1}$, where a_1 is the first term of the sequence.

43a. $\frac{3300}{3000} = 1.1; \frac{3630}{3300} = 1.1$
$a_4 = 3630 \cdot 1.1 = \$3993$
$a_5 = 3993 \cdot 1.1 = \$4392.30$

b. $\frac{3300}{3000} = 1.1; \frac{3630}{3300} = 1.1$
The common ratio is 1.1.

c. \$2727.27; divide tuition 3 years ago (\$3000) by 1.1, the common ratio.

TEST PREP

44. D: $\frac{10}{5} = 2$; $\frac{20}{10} = 2$; $\frac{40}{20} = 2$; there is a common ratio.

45. J; since $r = -4$ and $a_1 = 2$,
$\left(\frac{-8}{2} = -4;\ \frac{32}{-8} = -4;\ \frac{-128}{32} = -4\right)$
$a_n = 2\,(-4)^{n-1}$

46. C; $r = 2$ and $A_1 = 55$
$A_n = A_a r^{n-1}$
$A_7 = A_1\, r^6$
$A_7 = 3520$ Hz

CHALLENGE AND EXTEND

47. $\frac{x^2}{x} = x$; $\frac{x^3}{x^2} = x$
$r = x$ and $a_1 = x$;
$a_4 = x\,(x)^3 = x^4$
$a_5 = x\,(x)^4 = x^5$
$a_6 = x\,(x)^5 = x^6$

48. $\frac{6x^3}{2x^2} = 3x$; $\frac{18x^4}{6x^3} = 3x$
$r = 3x$ and $a_1 = 2x^2$;
$a_4 = 2x^2\,(3x)^3 = 54x^5$
$a_5 = 2x^2\,(3x)^4 = 162x^6$
$a_6 = 2x^2\,(3x)^5 = 486x^7$

49. $\frac{1}{y^2} \div \frac{1}{y^3} = y$; $\frac{1}{y} \div \frac{1}{y^2} = y$
$r = y$ and $a_1 = \frac{1}{y^3}$
$a_4 = \frac{1}{y^3}\,(y)^3 = 1$
$a_5 = \frac{1}{y^3}\,(y)^4 = y$
$a_6 = \frac{1}{y^3}\,(y)^5 = y^2$

50. $\frac{1}{x+1} \div \frac{1}{(x+1)^2} = x + 1$; $1 \div \frac{1}{x+1} = x + 1$
$r = x + 1$ and $a_1 = \frac{1}{(x+1)^2}$
$a_4 = \frac{1}{(x+1)^2}\,(x + 1)^3 = x + 1$
$a_5 = \frac{1}{(x+1)^2}\,(x + 1)^4 = (x + 1)^2$
$a_6 = \frac{1}{(x+1)^2}\,(x + 1)^5 = (x + 1)^3$

51. $a_{10} = a_1 r^9$
$a_1 = \frac{a_{10}}{r^9}$
$a_1 = \frac{0.78125}{(-0.5)^9}$
$a_1 = -400$

52. No; each term of the sequence is found by multiplying the previous term by the common ratio $\frac{1}{2}$. $\frac{1}{2}$ of any positive number is always another positive (nonzero) number.

53. $a_n = a_1 r^{n-1}$
$r^{n-1} = \frac{a_n}{a_1}$
$(0.4)^{n-1} = \frac{0.057344}{14}$
$(0.4)^{n-1} = (0.4)^6$
Then, $n - 1 = 6$
$n = 7$

54. Susanna assumed the sequence was geometric with $r = 2$. She used the formula to find $a_8 = 128$. Paul did not assume the sequence was geometric. Instead, he noticed a pattern of "add 1, add 2, and so on." He continued this pattern by adding 3, adding 4, etc., until he got the 8th term of 29. Both could be considered correct because it was not specified what type of sequence was given.

9-2 EXPONENTIAL FUNCTIONS

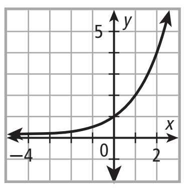
CHECK IT OUT!

1. $f(x) = 8(0.75)^x$
$f(3) = 8(0.75)^3$
$f(3) = 8(0.421875)$
$f(3) = 3.375$ in.

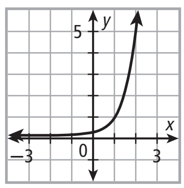
2a. No; as the x-values change by a constant amount, the y-values are not multiplied by a constant amount.

b. Yes; as the x-values change by a constant amount, the y-values are multiplied by a constant amount.

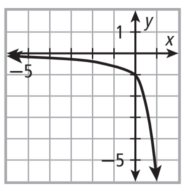
3a. $y = 2^x$



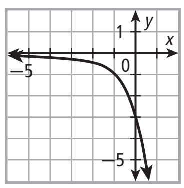
b. $y = 0.2(5)^x$



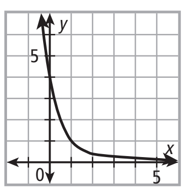
4a. $y = -6^x$



b. $y = -3(3)^x$



5a. $y = 4\left(\frac{1}{4}\right)^x$



b. $y = -2(0.1)^x$